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Kinematics and Dynamic Modeling for Holonomic Constrained Multiple Robot Systems through Principle of Workspace Orthogonalization

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This paper deals with an efficient mathematical modeling for multiple robot manipulators (or multifingered robot hands) holding and transporting a rigid common object on the constraint surfaces, subject to a set of holonomic (integrable) constraints. First, the kinematics and dynamics of the multiple robot systems are formulated. After a series of model transformations, a combined dynamic model is derived from a unified viewpoint. Then the system dynamics can be decomposed into two orthogonal subsystems: the (reduced-order) motion subsystem and the force subsystem. From a practical point of view, the new dynamic model presented in this paper is suitable form for dynamic analysis and hybrid (position/force) control synthesis.

Key Words: Multiple Robots, Common Object, Orthogonalization, Holonomic Constraints, Closed Kinematic Chains, Euler Angles, Contact Forces, Position (Force) Subsystems.

Nomenclature

$\Re^n$: The n-dimensional vector space with real elements $\Re$

$\Re^{n\times m}$: A set of all real-valued $(n\times m)$ matrices

$\mathbf{A}>0$: A positive definite matrix $\mathbf{A}$

$\|\mathbf{A}\|$: The induced norm of a real matrix $\mathbf{A}\in\Re^{n\times m}$; $\|\mathbf{A}\|=[\lambda_{\max}(\mathbf{A}^T\mathbf{A})]^{1/2}$ where $\lambda_{\max}$ is the maximum eigenvalue

C^p : A set of p-times continuously differentiable functions

$\mathbf{E}_n$: An $(n\times n)$ identity matrix

$\mathbf{0}_n$: A n-dimensional null vector

$\mathbf{0}_{n\times n}$: A $(n\times n)$ null matrices

$\dim(\circ)$: The dimension of $(\circ)$

$rs(\mathbf{A})$: The range space of matrix $\mathbf{A}$

$rk(\mathbf{A})$: The rank of matrix $\mathbf{A}$

$ns(\mathbf{A})$: The null space (or kernel) of a matrix $\mathbf{A}$

${}^a_i(\circ)_b$: a and b are arbitrary superscripts and subscripts, respectively, which are used to characterize the quantity $(\circ)$ of i th manipulator in the local coordinate system (Σ_b) with respect to a frame (Σ_a)

$\otimes$: The cross-product operation

$(\circ)^+$: The pseudoinverse or Moore-Penrose inverse of a matrix $(\circ)$

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1. Introduction

Recently, a single robotic system has been utilized in modern industries. Among the various approaches, the robust adaptive control methods (Sadegh, et. al., 1990; You, et. al., 1996) have been extensively used. Due to its capability and performance, the practical applications of such a system to higher level tasks are severely limited. To overcome the drawbacks, there has been growing interest in investigating the coordinated manipulation of multiple robot systems. For instance, in the advanced tasks involved in flexible manufacturing systems, the cooperation among two or more robots is required to accomplish the tasks successfully. Additionally, the multi-manipulator system provides higher flexibility and dexterity in performing complex tasks like human arms. Unfortunately, the multiple robot system forms closed kinematic chains, (Kiovo, et. al., 1992) which impose additional kinematic and dynamic constraints on the systems, thus the manipulation of such system is extremely complicated.

An important prerequisite to the dynamic analysis and the coordinated control of the multi-robot system is to obtain a proper mathematical model of such systems. The dynamics of constrained systems are discussed in references (Blajer, 1992; Huang, et. al., 1994; You, 1996). Further, the kinematics and dynamics of multiple robots are found in several places such as in references (Kiovo, et. al., 1990; Kerr, et. al., 1986; Chiacchio, et. al., 1991; Nakamura, 1991; Ahmad, et. al., 1991), among others. However, only a few articles consider the constrained multiple robot system carrying a common object. From a unified viewpoint, they have computationally inefficient formulations for position-force synthesis.

The major objective of this work is an attempt to develop a unified mathematical model for multi-fingered robots, cooperatively manipulating the common object along rigid constraint surfaces. First, the overall system model is obtained by combining the kinematics and dynamic constraints with the multi-manipulator dynamics. After a series of transformations, a reduced-order (decoupled) dynamic model is derived. Since the position- and force-controlled subsystems are efficiently decoupled in this method, each subsystem can be manipulated independently and simultaneously.

The content of this paper is organized as follows: In Sec. 2, the preliminaries and system description are presented. In Sec. 3, we provide the system parameters and kinematic formulations. Section. 4 gives the dynamics of manipulated object, while the unified dynamic model is proposed in Sec. 5. Finally, the conclusions of this paper are summarized in Sec. 6.

2. Preliminaries and System Description

Consider a cooperative multiple robot system in which multifingered hands are supposed to manipulate a common object along rigid constraint surfaces. As illustrated in Fig. 1, the multiple robot manipulators are constrained with each other as well as by the external environments. The overall system under consideration comprises three main components; $\nu(\geq 2)$ robotic manipulators, the common object (equivalently the payload), and the constraint surfaces. To facilitate further development, the following assumptions are made:

[A1]: Each robotic manipulator having n joints contacts the common object with a point, imposing the internal constraints.

[A2]: The constrained motions between the object and the external constraint surfaces are achieved through frictionless point contacts, imposing the external constraints.

[A3]: Each manipulator grasps the object firmly at a specified point, and their mutual positions and orientations are invariant throughout the system motions, i. e., a rigid grasping.

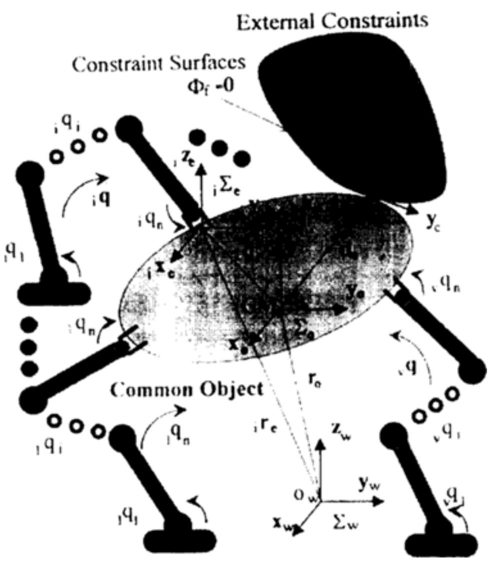

Fig. 1 Multiple robot manipulators carrying a common object on the constraint surfaces.

[A4]: The closed kinematic chains are always formed through the contact points.

3. System Parameters and Kinematic Formulations

This section is devoted to introducing the system variables and formulating the kinematic constraints, which result from the closed-chain mechanisms. To begin with, a set of coordinate systems are defined as follows (see Fig. 1). A frame $\sum_w\{o_w - x_w y_w z_w\}$ is the world (or absolute) coordinate system fixed to the ground as the reference frame, and its location generally depends on a task geometry to be performed; ${}_i\sum_e\{o_e - x_e y_e z_e\}_i$ is the i th end-effector coordinate whose origin ${}_io_e$ is assigned to the i th contact point between the end-effector and the common object; $\sum_o\{o_o - x_o y_o z_o\}$ is the common object frame, and its origin o_o is fixed to the mass center (CM); $\sum_c\{o_c - x_c y_c z_c\}$ is the constraint coordinate system whose origin o_c is located at the contact point between the object and the constraint surface. Unless mentioned otherwise, all Cartesian quantities are to be expressed in $\sum_w$. It is supposed that the index i takes all values from the integer set $[1, v]$ and indicates the quantity corresponding to the i th manipulator.

Some system variables are now defined. $\mathbf{p}_o = [\mathbf{r}_o^T\ \Omega_o^T] \in \Re^6$ is the generalized position vector representing the configuration of the common object, with $\mathbf{r}_o \in \Re^3$ and $\Omega_o = [\alpha_o \beta_o \gamma_o]^T \in \Re^3$. ${}_i\mathbf{p}_e = [{}_i\mathbf{r}_e^T\ {}_i\Omega_e^T] \in \Re^6$ denotes the generalized position vector of ith end-effector frame ${}_i\sum_e$, with ${}_i\mathbf{r}_e \in \Re^3$ and ${}_i\Omega_e = [{}_i\alpha_e\ {}_i\beta_e\ {}_i\gamma_e]^T$. The generalized position vector of $\sum_c$ relative to $\sum_w$ is given by $\mathbf{p}_c = [\mathbf{r}_c^T\ \Omega_c^T]$, with $\mathbf{r}_c \in \Re^3$ and $\Omega_c = [\alpha_c\ \beta_c\ \gamma_c]^T$. The vector ${}_i\mathbf{d} \in \Re^3$ denotes the distance from the CM (o_o) to each contact point ${}_io_e$ measured in $\sum_o$, while the distance between o_o and o_c in terms of $\sum_o$ is specified by $\mathbf{d}_o \in \Re^3$.

In addition, ${}_i\mathbf{F}_e = [{}_i\mathbf{f}_e^T\ {}_i\mathbf{n}_e^T]^T \in \Re^6$ denotes the vector of generalized end-effector forces (or wrenches) acting through the contact point ${}_io_e$ to the common object, where ${}_i\mathbf{f}_e \in \Re^3$ and ${}_i\mathbf{n}_e \in \Re^3$ are the vectors of the forces and the torques, respectively. $\mathbf{F}_o = [\mathbf{f}_o^T\ \mathbf{n}_o^T]^T \in \Re^6$ represents the generalized forces acting on the CM of the object by ${}_i\mathbf{F}_e$. In this paper, the term generalized positions include both positions and orientations, and the generalized forces include both forces and torques.

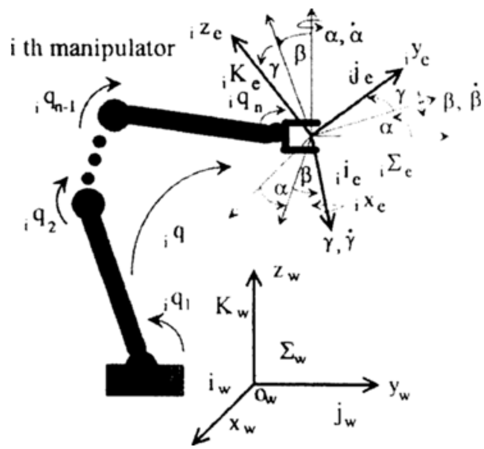

Fig. 2 Geometrical representation of Euler angles (yaw, pitch, and roll).

For representing an arbitrary orientation of a rigid body in the space $\Re^3$, the rotational motion ${}^a\Omega_b$ can be described by three Euler angles shown pictorially in Fig. 2. More specifically, the Euler angles are specified in terms of the images of three parameters $\{\alpha, \beta, \text{and } \gamma\}$, obtained by performing three elementary rotations of a body-attached frame $\sum_b$ (where ${}_i\sum_e$, $\sum_o$, and $\sum_c$) with respect to the fixed frame $\sum_a$ (where $\sum_w$); that is, rotating α angle about the z axis first, then β angle about the new y axis, and finally γ angle about the new x axis. Then the resulting overall transformation with the Euler angles is given in a 3×3 matrix as

$$ {}^w_i\mathbf{R}_e = \begin{bmatrix} C_\alpha C_\beta & C_\alpha S_\beta S_\gamma - S_\alpha C_\gamma & C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma \\ S_\alpha C_\beta & S_\alpha S_\beta S_\gamma + C_\alpha C_\gamma & S_\alpha S_\beta C_\gamma - C_\alpha S_\gamma \\ -S_\beta & C_\beta S_\gamma & C_\beta C_\gamma \end{bmatrix} \quad (1) $$

where $c_\alpha = \cos(\alpha)$, $s_\beta = \sin(\beta)$, and $c_\gamma = \cos(\gamma)$, and so on. In Fig. 2, the vectors $\mathbf{i}_{(\circ)}$, $\mathbf{j}_{(\circ)}$, and $\mathbf{k}_{(\circ)} \in \Re^3$ denote coordinate vectors of the principle axes of the arbitrary frame $\sum_{(\circ)}$ (where ${}_i\sum_e$ or $\sum_w$). Thus the orthogonal rotation matrix ${}^w\mathbf{R}_e(\circ) : \Re^3 \rightarrow \Re^3$ (with $\mathbf{R}\mathbf{R}^T = \mathbf{E}$ and $\mathbf{R}^{-1} = \mathbf{R}^T$) maps the vectors in $\sum_b$ to those in $\sum_w$.

Furthermore, ${}^a\omega_b \in \Re^3$ describes the vector of the angular velocity of a frame $\sum_b$ as viewed in $\sum_a$. The generalized motion (You, 1994) is specified by $(\mathbf{r}, \mathbf{R}) \in \Re^3 \times SO(3) = SE(3)$, where $SO(3)$ denotes a set of all proper 3×3 rotation matrices on $\Re^3$, which is a three-dimensional submanifold of $\Re^9$. It can be formally defined as the Special Orthogonal group of order of 3:

$$SO(3) = \{\mathbf{R} \in \Re^{3\times3};\ \det(\mathbf{R}) = 1,\ \mathbf{R}^T\mathbf{R} = \mathbf{E}\}.$$

Consequently, the motions of the rigid body belonging to 6-dimensional manifold are represented by the Special Euclidean group $SE(3)$. The rotational velocity representing the relationship between the angular velocity (${}^a\omega_b$) and the rates of Euler angles (${}^a\dot{\Omega}_b = [\dot{\alpha}\ \dot{\beta}\ \dot{\gamma}]^T$) is given by

$${}^a\omega_b = {}^a\Gamma_b{}^a\dot{\Omega}_b \tag{2}$$

where the mapping function $\Gamma \in \Re^{3\times3}$ is defined as (Amirouche, 1992; You, 1996)

$$\Gamma(\Omega) = \begin{bmatrix} 0 & -s_\alpha & c_\beta c_\alpha \\ 0 & c_\alpha & c_\beta s_\alpha \\ 1 & 0 & -s_\beta \end{bmatrix}$$

The singularity (or degeneracy) is likely to occur at $\det(\Gamma) = 0$ in which the Jacobian Γ is rank deficient. Although the singularity is not avoided in Euler angle representations, the matrix Γ is assumed to be nonsingular over any Ω of interest.

With the notations defined above, the two representations of the velocities (or twists) are related as

$${}^a\mathbf{v}_b = {}^a\mathbf{N}_b{}^a\dot{\mathbf{p}}_b \tag{3}$$

where ${}^a\mathbf{v}_b = [{}^a\dot{\mathbf{r}}_b^T\ {}^a\omega_b^T]^T \in \Re^6$ and ${}^a\dot{\mathbf{p}}_b = [{}^a\dot{\mathbf{r}}_b^T\ {}^a\dot{\Omega}_b^T]^T \in \Re^6$. A nonsingular matrix ${}^a\mathbf{N}_b$ is given by ${}^a\mathbf{N}_b = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & {}^a\Gamma_b \end{bmatrix} \in \Re^{6\times6}$.

Let ${}_i\mathbf{q} = [{}_iq_1 \cdots {}_iq_n]^T$ be the vector of the joint positions for the i th manipulator. Then the position vectors are suitably arranged to form the extended joint-space variables $\mathbf{q}_s \in \Re^{vn}$ with $\mathbf{q}_s = [{}_1\mathbf{q} \cdots {}_v\mathbf{q}]^T$. Each manipulator has a forward kinematics providing the relationships between the joint-space and the operational space variables as

$${}_i\mathbf{p}_e = {}_i\mathbf{h}({}_i\mathbf{q}),\ (i = 1, \cdots, v) \tag{4}$$

where ${}_i\mathbf{h}(\circ) : C^2(\Re^n \rightarrow \Re^6)$ is twice continuously differentiable function. By virtue of Eq. (3), the twist vector of the end-effector for the ith manipulator is analogously given as

$${}_i\mathbf{v}_e = {}_i\mathbf{N}_e\ {}_i\dot{\mathbf{p}}_e = {}_i\mathbf{J}\,{}_i\dot{\mathbf{q}} \tag{5}$$

where ${}_i\mathbf{v}_e = [{}_i\dot{\mathbf{r}}_e^T\ {}_i\omega_e^T]^T$ and ${}_i\dot{\mathbf{p}}_e = [{}_i\dot{\mathbf{r}}_e^T\ {}_i\dot{\Omega}_e^T]^T$, with ${}_i\mathbf{N}_e = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & {}_i\Gamma_e \end{bmatrix}$. The matrix ${}_i\mathbf{J} = {}_i\mathbf{N}_e \frac{\partial_i\mathbf{h}}{\partial_i\mathbf{q}} \in \Re^{6\times n}$, which is the standard Jacobian of the i th manipulator with a full rank, transforms the vector of joint-space velocities to that of the end-effector velocities with $\det({}_i\mathbf{J}) \neq 0$. Due to rigid grasping, there are no relative motions between the object and the arm's end-effectors. The following kinematic relation can be established at each contact point

$${}_i\mathbf{r}_e = \mathbf{r}_o + {}^w\mathbf{R}_o\ {}_i\mathbf{d} \tag{6a}$$

$${}_i\omega_e = \omega_o \tag{6b}$$

where $\mathbf{R} \in SO(3)$ $(\subset \Re^{3\times3})$ is an orthogonal rotation matrix that transforms the local vectors in $\sum_o$ to the representations in $\sum_w$. The following identities can be utilized for the further development:

$$\frac{d}{dt}(\mathbf{R}) = \omega\otimes\mathbf{R} \text{ and } \omega\otimes\mathbf{x} = -\mathbf{x}\otimes\omega,\ \forall\mathbf{x}\in\Re^3$$

With the foregoing definitions, the following equation relates the generalized velocity of the end-effector to that of object's center of mass:

$${}_i\mathbf{v}_e = {}_i\mathbf{Q}\mathbf{N}_o\dot{\mathbf{p}}_o = {}_i\mathbf{Q}^T\mathbf{v}_o \tag{7}$$

where $\mathbf{v}_o = [\dot{\mathbf{r}}_o^T\ \omega_o^T]^T \in \Re^6$ and $\dot{\mathbf{p}}_o = [\dot{\mathbf{r}}_o^T\ \dot{\Omega}_o^T]^T \in \Re^6$, with $\mathbf{N}_o = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \Gamma_o \end{bmatrix}$. The mapping function ${}_i\mathbf{Q} \in \Re^{6\times6}$ is defined as

$${}_i\mathbf{Q} = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0}_{3\times3} \\ -\mathbf{D}({}^w\mathbf{R}_o\ {}_i\mathbf{d}) & \mathbf{E}_3 \end{bmatrix} \tag{8}$$

which is the positive-definite and nonsingular matrix. In Eq. (8), the operator $\mathbf{D} \in \Re^{3\times3}$ is introduced as

$$\mathbf{D}(\mathbf{z}) = \mathbf{z}\otimes, \text{ with } \mathbf{z}\in\Re^3$$

where

$$\mathbf{D}(\mathbf{z}): \Re^3 \rightarrow so(3) \left| \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right| \mapsto \begin{bmatrix} 0 & -z_3 & z_2 \\ z_3 & 0 & -z_1 \\ -z_2 & z_1 & 0 \end{bmatrix}.$$

Thus the matrix $\mathbf{D}$ identifies a one-to-one correspondence between a three-dimensional vector $\mathbf{z}$ and $so(3)$, which is a 3×3 skew-symmetric matrix with $so(3) = \{\mathbf{A} \in \Re^{3\times3}; \mathbf{A}^T = -\mathbf{A}\}$.

Aggregating all robots acting on the common object gives

$$\mathbf{v}_e = \tilde{\mathbf{N}}\dot{\mathbf{p}}_e = \mathbf{J}_s\mathbf{v}_o = \mathbf{Q}_s^T\mathbf{v}_o \tag{9}$$

where all terms can be augmented as

$$\mathbf{v}_e = [{}_1\mathbf{v}_e^T \cdots {}_v\mathbf{v}_e^T]^T \in \Re^{6v}$$
$$\dot{\mathbf{p}}_e = [{}_1\dot{\mathbf{p}}_e^T \cdots {}_v\dot{\mathbf{p}}_e^T]^T \in \Re^{6v}$$
$$\dot{\mathbf{q}}_s = [{}_1\dot{\mathbf{q}}^T \cdots {}_v\dot{\mathbf{q}}^T]^T \in \Re^{vn}$$
$$\tilde{\mathbf{N}} = Block\ diag[{}_1\mathbf{N}_e, \cdots, {}_v\mathbf{N}_e],\ \tilde{\mathbf{N}} \in \Re^{6v\times6v}$$
$$\mathbf{J}_s = Block\ diag[{}_1\mathbf{J}, \cdots, {}_v\mathbf{J}],\ \mathbf{J}_s \in \Re^{6v\times6v}$$
$$\mathbf{Q}_s = [{}_1\mathbf{Q} \cdots {}_v\mathbf{Q}] \in \Re^{6\times6v}$$

In the above formula, the operator $\mathbf{Q}_s$, which has a full row rank with $rk(\mathbf{Q}_s) = 6$, is called the "grasp" matrix. (Chiacchio, et. al., 1991; You, 1994)

Similarly, the generalized position vector of o_c relative to $\sum_w$ is given by

$$\mathbf{r}_c = \mathbf{r}_o + {}^w\mathbf{R}_o\mathbf{d}_o \tag{10a}$$
$$\omega_c = \omega_o \tag{10b}$$

The corresponding velocity constraint on the external surfaces is obtained as

$$\mathbf{v}_c = \mathbf{Q}_o^T\mathbf{v}_o \tag{11}$$

where, by the virtue of (8), $\mathbf{Q}_o$ is defined as

$$\mathbf{Q}_o = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0}_{3\times3} \\ -\mathbf{D}({}^w\mathbf{R}_o\mathbf{d}_o) & \mathbf{E}_3 \end{bmatrix} \in \Re^{6\times6}.$$

4. Dynamics of Manipulated Object

First, consider the common object system in which the object is rigidly grasped by v robotic arms without the external constraints. We can describe the dynamics of a manipulated object in $\sum_w$ as follows:

$$\bar{m}_o\ddot{\mathbf{r}}_o + \bar{m}_o\mathbf{g} = \mathbf{f}_o \tag{12a}$$
$${}^w\mathbf{R}_o\bar{\mathbf{I}}\,{}^w\mathbf{R}_o^T\dot{\omega}_o + \omega_o \otimes [{}^w\mathbf{R}_o\bar{\mathbf{I}}\,{}^w\mathbf{R}_o^T\omega_o] = \mathbf{n}_o \tag{12b}$$

where $\bar{m}_o \in \Re^+$ and $\bar{\mathbf{I}} \in \Re^{3\times3}$ represent the object's mass and the inertia matrix, respectively. $\mathbf{g} = [0\ 0\ -9.8]^T$ denotes the vector of gravitational accelerations. The wrenches ($\mathbf{f}_o \in \Re^3$ and $\mathbf{n}_o \in \Re^3$), representing the vector of the resultant forces applied to the CM of the object by v manipulators through the contact points, are defined as

$$\mathbf{f}_o = \Sigma_{i=1}^{v}{}_i\mathbf{f}_e \tag{13a}$$
$$\mathbf{n}_o = \Sigma_{i=1}^{v}({}_i\mathbf{n}_e - {}^w\mathbf{R}_{oi}\mathbf{d} \otimes {}_i\mathbf{f}_e) \tag{13b}$$

The object dynamics (12a, b) can be put into a compact matrix-vector form as

$$\mathbf{M}_o\dot{\mathbf{v}}_o + \mathbf{C}_o\mathbf{v}_o + \mathbf{G}_o = \mathbf{F}_o \tag{14}$$

where

$$\dot{\mathbf{v}}_o = [\ddot{\mathbf{r}}_o^T\dot{\omega}_o^T]^T \in \Re^6$$
$$\mathbf{M}_o = \begin{bmatrix} \bar{m}_o\mathbf{E}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & {}^w\mathbf{R}_o\bar{\mathbf{I}}\,{}^w\mathbf{R}_o^T \end{bmatrix} \in \Re^{6\times6}$$
$$\mathbf{C}_o = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{D}(\omega_o)\,{}^w\mathbf{R}_o\bar{\mathbf{I}}\,{}^w\mathbf{R}_o^T \end{bmatrix} \in \Re^{6\times6}$$
$$\mathbf{G}_o = \begin{bmatrix} \bar{m}_o\mathbf{g} \\ \mathbf{0}_3 \end{bmatrix} \in \Re^6,$$
$$\mathbf{F}_o = [\mathbf{f}_o^T\ \mathbf{n}_o^T]^T \in \Re^6.$$

It should be noted that the dynamic model (14) satisfies the following fundamental properties (Ahmad, et. al., 1991; You, 1994).

[P1]: $\mathbf{M}_o$ is a symmetric and positive-definite inertia matrix.

[P2]: $(\dot{\mathbf{M}}_o - 2\mathbf{C}_o)$ is a skew-symmetric matrix.

From Eqs. (13a, b), the equivalent forces on the object can be further characterized as

$$\mathbf{F}_o = \Sigma_{i=1}^{v}{}_i\mathbf{Q}_i\mathbf{F}_e = \mathbf{Q}_s\mathbf{F}_s \tag{15}$$

with $\mathbf{F}_o = [{}_1\mathbf{F}_e^T \cdots {}_v\mathbf{F}_e^T]^T \in \Re^{6v}$. By virtue of the duality between the force (wrench) and the velocity (twist), we can also determine the wrench (15) by referring to the Eq. (9). In this case, the grasp matrix $\mathbf{Q}_s$ is referred to as the force transmission matrix, which is to identify the contributions of the interaction forces of each manipulator to the external forces on the CM of the object. For the frictionless point contacts, the number of independent constraints by rigid grasping is equal to $m_c(=6v - rk(\mathbf{Q}_s))$.

We now turn to the problem of considering the external constraints imposed on the common object. If the constraint surfaces have dimension $m(<6)$, the algebraic equations can be expressed as (You, 1996)

$$\Phi_f(\mathbf{p}_o) = [\phi_{f1}(\mathbf{p}_o) \cdots \phi_{fm}(\mathbf{p}_o)]^T = \mathbf{0}_m \quad \text{with } \mathbf{p}_o \in SE(3) \tag{16}$$

where $\Phi_f(\circ)$: $C^2(\Re^6 \to \Re^m)$ is a differentiable and mutually independent function over any $\mathbf{p}_o$ of interest in a subset of $\Re^6$. The "natural" restrictions given by (16) are commonly called "holonomic" constraints. (Amirouche, 1992; You, 1994). The corresponding velocity constraints can be obtained by

$$\mathbf{J}_f \mathbf{v}_o = \mathbf{0} \tag{17}$$

where

$$\mathbf{J}_f = \frac{\partial \Phi_f}{\partial \mathbf{p}_o} = \left[\frac{\partial \phi_{f1}}{\partial \mathbf{p}_o}^T \cdots \frac{\partial \phi_{fm}}{\partial \mathbf{p}_o}^T \right]^T \in \Re^{m \times k}$$

with $\frac{\partial \phi_{fi}}{\partial \mathbf{p}_o}^T \in \Re^6$.

Since the above geometric constraints are mutually independent, the matrix $\mathbf{J}_f$ has a full row rank with $rk(\mathbf{J}_f) = m$. Note that the row vectors of $\mathbf{J}_f$ span the normal space of the constraint surfaces. Provided that the common object is constrained to follow the rigid constraint surfaces, the system is also subject to a set of $(6-m)$ "artificial" constraints, namely,

$$\Phi_p(\mathbf{p}_o) = [\varphi_{p1}(\mathbf{p}_o) \cdots \varphi_{p(6-m)}(\mathbf{p}_o)]^T \tag{18}$$

where $\Phi_p(\circ)$: $C^2(\Re^6 \rightarrow \Re^{(6-m)})$ is a mutually independent function. Evidently, the combined sets $\{\Phi_f, \Phi_p\}$ are mutually independent and twice differentiable with respect to $\mathbf{p}_o$ such that the constraint surfaces can be parameterized by

$$\mathbf{p}_c = [\Phi_f^T \ \Phi_p^T]^T \in \Re^6 \tag{19}$$

The velocity relation can be obtained as

$$\mathbf{v}_c = \mathbf{J}_{pf} \mathbf{v}_o \tag{20}$$

with $\mathbf{J}_{pf} = [\mathbf{J}_f^T \mathbf{J}_p^T]^T \in \Re^{6\times 6}$. We can define the matrix $\mathbf{J}_p \in \Re^{(6-m)\times 6}$, with $rk(\mathbf{J}_p) = 6-m$, whose row vectors span the tangent space of the constraint surfaces as

$$\mathbf{J}_p = \frac{\partial \Phi_p}{\partial \mathbf{p}_o} = \left[\frac{\partial \varphi_{p1}}{\partial \mathbf{p}_o}^T \cdots \frac{\partial \varphi_{p(6-m)}}{\partial \mathbf{p}_o}^T \right]^T$$

with $\frac{\partial \varphi_{pj}}{\partial \mathbf{p}_o}^T \in \Re^6$

Then the following relations hold:

$$\mathbf{J}_p \cdot \mathbf{J}_f^T = \mathbf{0} \ (\text{or } \mathbf{J}_f \cdot \mathbf{J}_p^T = \mathbf{0})$$

It is clear that the column vectors of $\mathbf{J}_f^T$ span the null space of $\mathbf{J}_p$ (i. e., $rs(\mathbf{J}_f^T) \subseteq ns(\mathbf{J}_p)$). In addition, the vector space spanned by $\mathbf{J}_f^T$ is the space of all vectors orthogonal to $\mathbf{J}_p$ and called the orthogonal complement of $\mathbf{J}_p$. As a consequence, the constraint surface frame has a set of vectors

$$\left\{ \frac{\partial \phi_{fi}}{\partial \mathbf{p}_o}^T, \ (i=1, \cdots, m) \ ; \ \frac{\partial \varphi_{pj}}{\partial \mathbf{p}_o}^T, \ (j=1, \cdots, (6-m)) \right\}$$

as basis in 6-dimensional spaces.

Remark 1 {Direct sum and decomposition}: Let $rs(\mathbf{J}_f^T)$ and $rs(\mathbf{J}_p^T)$ be the two subspaces of $\Re^6$. In case $rs(\mathbf{J}_f^T) \cap rs(\mathbf{J}_p^T) = \{\mathbf{0}\}$, the space $\{rs(\mathbf{J}_f^T) + rs(\mathbf{J}_p^T)\}$ is referred to as the direct sum of the subspaces $rs(\mathbf{J}_f^T)$ and $rs(\mathbf{J}_p^T)$ and represented by $rs(\mathbf{J}_f^T) \oplus rs(\mathbf{J}_p^T)$. Further, one can obtain the following result:

$$\dim\{rs(\mathbf{J}_f^T) \oplus rs(\mathbf{J}_p^T)\} = \dim\{rs(\mathbf{J}_f^T)\} + \dim\{rs(\mathbf{J}_p^T)\}.$$

Based on the remark, it is possible to decompose a given position vector into two orthogonal subspaces

$$\Re^6 = rs(\mathbf{J}_f^T) \oplus rs(\mathbf{J}_p^T), \text{ with}$$
$$rs(\mathbf{J}_f^T) \cap rs(\mathbf{J}_p^T) = \{\mathbf{0}\}.$$

In other words, the $rs(\mathbf{J}_p^T)$ specifies the motion subspace, and the $rs(\mathbf{J}_f^T)$ spans the force-controlled subspace.

Then the resultant forces $\mathbf{F}_{oc} \in \Re^6$ at the CM exerted by the contact forces can be obtained by

$$\mathbf{F}_{oc} = \mathbf{J}_f^T \sigma \tag{21}$$

where $\sigma \in \Re^m$ is the vector of Lagrange multipliers associated with m constraint surfaces. Finally, the dynamics of the manipulated object can be given by

$$\mathbf{M}_o \dot{\mathbf{v}}_o + \mathbf{C}_o \mathbf{v}_o + \mathbf{G}_o = \mathbf{Q}_s \mathbf{F}_s - \mathbf{J}_f^T \sigma \tag{22}$$

The object model is now combined with the dynamic model of multifingered robot hands via the constraint equations to formulate the dynamics of the entire system.

5. A Unified System Modeling of Constrained Multiple Robots

Using Lagrange's formulation, the dynamic model for the i th rigid robot is described in the joint-space variables as

$$ {}_i\mathbf{M}({}_i\mathbf{q};\ {}_i\Theta)\,{}_i\ddot{\mathbf{q}} + {}_i\mathbf{C}({}_i\mathbf{q},\ {}_i\dot{q};\ {}_i\Theta)\,{}_i\dot{\mathbf{q}} + {}_i\mathbf{G}({}_i\mathbf{q};\ {}_i\Theta) = {}_i\mathbf{T} - {}_i\mathbf{J}^T({}_i\mathbf{q})\,{}_i\mathbf{F}_e \qquad (23) $$

where ${}_i\mathbf{q}$, ${}_i\dot{\mathbf{q}}$, and ${}_i\ddot{\mathbf{q}}$ are the generalized vectors representing the joint positions, velocities, and accelerations, respectively, ${}_i\mathbf{M}\in\Re^{n\times n}$ is an inertia matrix, ${}_i\mathbf{C}\in\Re^{n\times n}$ is a matrix function containing terms such as Coriolis and centripetal torques, ${}_i\mathbf{G}\in\Re^n$ is the vector of gravity torques, ${}_i\mathbf{T}\in\Re^n$ is the joint torque vector, and ${}_i\Theta\in\Re^{k_1}$ is the vector of manipulator parameters (e. g., link masses, link lengths, moments of inertia). All other terms are previously defined.

It is known that the dynamics of the individual manipulator (23) has the following fundamental properties (Sadegh, et. al., 1990; You, et. al., 1996):

[P3]: ${}_i\mathbf{M}$ is a symmetric and positive-definite matrix, i. e., ${}_i\mathbf{M}={}_i\mathbf{M}^T>0$.

[P4]: $({}_i\dot{\mathbf{M}}-2{}_i\mathbf{C})$ is a skew-symmetric matrix, that is, $\mathbf{x}^T({}_i\dot{\mathbf{M}}-2{}_i\mathbf{C})\mathbf{x}=\mathbf{0}$, $\forall x\in\Re^n$.

[P5]: A part of the dynamics (23) is linear in terms of a suitably defined set of parameters,

$$ {}_i\mathbf{M}({}_i\mathbf{q};\ {}_i\Theta)\mathbf{y} + {}_i\mathbf{C}({}_i\mathbf{q},\ {}_i\dot{\mathbf{q}};\ {}_i\Theta)\mathbf{x} + {}_i\mathbf{G}({}_i\mathbf{q};\ {}_i\Theta) = {}_i\mathbf{Y}({}_i\mathbf{q},\ {}_i\dot{\mathbf{q}},\ \mathbf{x},\ \mathbf{y})\,{}_i\Theta, $$

where ${}_i\mathbf{Y}\in\Re^{n\times k_1}$ is a regressor matrix (You, et. al., 1996) with some vectors $({}_i\mathbf{q},\ {}_i\dot{\mathbf{q}},\ \mathbf{x},\ \mathbf{y})\in\Re^n$, and ${}_i\Theta\in\Re^{k_1}$ is the vector of system parameters of interest.

The extended joint-space dynamics obtained by grouping v such equations can be expressed in a concise form as

$$ \mathbf{M}_s(\mathbf{q}_s;\ \Theta)\,\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s,\ \dot{\mathbf{q}}_s;\ \Theta)\,\dot{\mathbf{q}}_s + \mathbf{G}_s(\mathbf{q}_s;\ \Theta) = \mathbf{T}_s - \mathbf{J}_s^T(\mathbf{q}_s)\,\mathbf{F}_s \qquad (24) $$

where all terms are compacted into

$\mathbf{q}_s=[{}_1\mathbf{q}^T\cdots{}_v\mathbf{q}^T]^T\in\Re^{vn}$,
$\mathbf{M}_s=$ *Block diag*$[{}_1\mathbf{M}({}_1\mathbf{q}),\cdots,{}_v\mathbf{M}({}_v\mathbf{q})]$, $\mathbf{M}_s\in\Re^{vn\times vn}$
$\mathbf{C}_s=$ *Block diag*$[{}_1\mathbf{C},\cdots,{}_v\mathbf{C}]$, $\mathbf{C}_s\in\Re^{vn\times vn}$
$\mathbf{G}_s=[{}_1\mathbf{G}^T\cdots{}_v\mathbf{G}^T]^T\in\Re^{vn}$,
$\mathbf{J}_s=$ *Block diag*$[{}_1\mathbf{J},\cdots,{}_v\mathbf{J}]$, $\mathbf{J}_s\in\Re^{6v\times vn}$
$\mathbf{F}_s=[{}_1\mathbf{F}_e^T\cdots{}_v\mathbf{F}_e^T]^T\in\Re^{6n}$,
$\mathbf{T}_s=[{}_1\mathbf{T}^T\cdots{}_v\mathbf{T}^T]^T\in\Re^{vn}$,
$\Theta\in\Re^{k_2}$.

The physical meanings of all terms above have been previously explained. Notice that the dynamics (24) also satisfies the fundamental properties listed in (23). Although the kinematic redundancies are important to the development of more dexterous robot systems, we focus our attention only to kinematically non-redundant robotic arms (i. e., $n=6$).

The object dynamics will be transformed into the formulations in the extended joint-space in what follows. From (9), the twist vector for the common object in the Cartesian space can be written as

$$ \mathbf{v}_O=\mathbf{J}_{sL}\dot{\mathbf{q}}_s \qquad (25) $$

where $\mathbf{J}_{sL}=(\mathbf{Q}_s^T)^+\mathbf{J}_s$, with $\mathbf{J}_{sL}\in\Re^{6\times 6v}$ and $(\mathbf{Q}_s^T)^+\in\Re^{6\times 6v}$. The matrix $(\mathbf{Q}_s)^+$ with $\mathbf{Q}_s$ having a full rank is defined as

$$ (\mathbf{Q}_s)^+=\mathbf{Q}_s^T[\mathbf{Q}_s\mathbf{Q}_s^T]^{-1},\ \text{with}\ \mathbf{E}=\mathbf{Q}_s(\mathbf{Q}_s)^+ $$

where $(\mathbf{Q}_s)^+$ satisfies the four Penrose conditions (You, 1994). Furthermore, differentiating (25) with respect to time yields

$$ \dot{\mathbf{v}}_O=\dot{\mathbf{J}}_{sL}\dot{\mathbf{q}}_s+\mathbf{J}_{sL}\ddot{\mathbf{q}}_s \qquad (26) $$

From (22), the object dynamics can be rewritten as

$$ \mathbf{M}_O\dot{\mathbf{v}}_O+\mathbf{C}_O\mathbf{v}_O+\mathbf{G}_O+\mathbf{J}_f^T\sigma=\mathbf{Q}_s\mathbf{F}_s=\mathbf{F}_{Os} \qquad (27) $$

where $\mathbf{F}_{Os}\in\Re^6$ represents the vector of resultant forces on the common object. For the given forces $\mathbf{F}_{Os}$, the general solution of Eq. (27) can be obtained in the form

$$ \mathbf{F}_s=(\mathbf{Q}_s)^+\mathbf{F}_{Os}+\mathbf{S}\mathbf{f}_{cI} \qquad (28) $$

where $\mathbf{S}\in\Re^{6v\times m_c}$ and $\mathbf{f}_{cI}\in\Re^{m_c}$, with $rk(\mathbf{S})=m_c$ and $rs(\mathbf{S})\subset rs(\mathbf{E}_{6v}-(\mathbf{Q}_s)^+\mathbf{Q}_s)$. Since the matrix $\mathbf{S}$ is the orthogonal complement to $\mathbf{Q}_s$ (or $\mathbf{Q}_s\cdot\mathbf{S}=\mathbf{0}$), the operator $\mathbf{S}$ projects any arbitrary vector $\mathbf{f}_{cI}$ into the null space of $\mathbf{Q}_s$. We note that the choice for $\mathbf{f}_{cI}$ is not unique, however, $\mathbf{S}\mathbf{f}_{cI}$ lies in the null space of $\mathbf{Q}_s$, where $rs(\mathbf{S})\subset ns(\mathbf{Q}_s)$. Due to the kinematically redundant mapping between the end-effector space and the object space (that is, $\dim(\mathbf{F}_s)>\dim(\mathbf{F}_{Os})$), there exist an infinite number of solutions for the end-effector forces to provide $\mathbf{F}_{Os}$. The first term of (28), denoted by $(\mathbf{Q}_s)^+\mathbf{F}_{Os}$, is called the minimum (Euclidean) norm solution and is the component of $\mathbf{F}_s$ that contributes the motion of object $\mathbf{F}_{Os}$. The other term, denoted by $\mathbf{S}\mathbf{f}_{cI}$, is referred to the null solution and is the subspace of the forces $\mathbf{F}_s$ which causes the internal (or grasping) forces on the object. The internal forces do not affect any motion of object. In fact, the manipulation forces and internal forces can be determined by using the column spaces and the null spaces of $\mathbf{Q}_s$, respectively.

Consider the motion constraints of the overall system resulting from physical contacts. To begin with, the external constraints between the common object and the rigid constraint surfaces are investigated. The constraint Eq. (17) can be expressed in the extended joint-space variables as

$$\mathbf{J}_f\mathbf{J}_{sL}\dot{\mathbf{q}}_s=\mathbf{0}_m \tag{29}$$

In addition, there exist the internal constraints between the end-effectors and the common object. Suppose that the constraint equations in the end-effector variables are given by

$$\Psi_f(\mathbf{p}_e)=\mathbf{0}_{m_c},\ \text{with}\ \mathbf{p}_e\in SE(3) \tag{30}$$

where $\Psi_f(\circ)$: $C^2(\Re^{6v}\rightarrow\Re^{m_c})$ is $m_c(<6v)$ mutually independent and differentiable function. Then the velocity constraints can be written as

$$\mathbf{S}^T\mathbf{v}_e=\mathbf{S}^T\mathbf{J}_s\dot{\mathbf{q}}_s=\mathbf{0}_{m_c},\ \text{with}\ \mathbf{S}^T=\frac{\partial\Psi_f}{\partial\mathbf{p}_e} \tag{31}$$

Next, combining Eqs. (29) and (31) yields

$$\mathbf{J}_{fI}\dot{\mathbf{q}}_s=\mathbf{0}_{m_t},\ \text{with}\ \mathbf{J}_{fI}=\begin{bmatrix}\mathbf{S}^T & \mathbf{J}_s\\ \mathbf{J}_f & \mathbf{J}_{sL}\end{bmatrix}\in\Re^{m_t\times 6v} \tag{32}$$

where $m_t(=m+m_c)$ is the number of total contact constraints, namely, the dimension of force-manipulated subspaces. Thus the constraint Jacobian matrix $\mathbf{J}_{fI}$ with a full rank projects the joint-space velocities into the normal directions of a set of hypersurfaces described by $\Psi_f=\mathbf{0}$ and $\Phi_f=\mathbf{0}$.

Based on the above observations, the combined contact forces can be defined as

$$\mathbf{F}_{cI}=[\mathbf{f}_{cI}{}^T\sigma^T]^T\in\Re^{m_t} \tag{33}$$

The corresponding forces can be expressed in the joint-space as

$$\mathbf{T}_{cI}=\mathbf{J}_{fI}{}^T\mathbf{F}_{cI}. \tag{34}$$

It is worth noting that the number of degrees of freedom (DOF) lost in motion due to the closed kinematic chains is equal to the dimension of the spaces for the total contact forces.

The kinematic and dynamic constraints are then combined with the extended manipulator dynamics to formulate the entire mathematical model. To do this, introducing (25) through (27) into (28) and substituting the resulting equation into (24) yields

$$\mathbf{M}_s\ddot{\mathbf{q}}_s+\mathbf{C}_s\dot{\mathbf{q}}_s+\mathbf{G}_s+\mathbf{J}_s{}^T\{(\mathbf{Q}_s)^+[\mathbf{M}_o\dot{\mathbf{v}}_o+\mathbf{C}_o\mathbf{v}_o+\mathbf{G}_o+\mathbf{J}_f{}^T\sigma]+\mathbf{S}\mathbf{f}_{cI}\}=\mathbf{T}_s.$$

After some algebraic manipulations using (32) and (33) with $\mathbf{J}_s{}^T(\mathbf{Q}_s)^+=\mathbf{J}_{sL}{}^T$, the above equation can be abbreviated in a concise matrix-vector form as

$$\mathbf{M}_u\ddot{\mathbf{q}}_s+\mathbf{C}_u\dot{\mathbf{q}}_s+\mathbf{G}_u=\mathbf{T}_s-\mathbf{J}_{fI}{}^T\mathbf{F}_{cI} \tag{35}$$

where the effective quantities in the extended joint-space are given as

$$\mathbf{M}_u=\mathbf{M}_s+\mathbf{J}_{sL}{}^T\mathbf{M}_o\mathbf{J}_{sL},\ M_u\in\Re^{6v\times 6v}$$
$$\mathbf{C}_u=\mathbf{C}_s+\mathbf{J}_{sL}{}^T\mathbf{M}_o\dot{\mathbf{J}}_{sL}+\mathbf{J}_{sL}{}^T\mathbf{C}_o\mathbf{J}_{sL},$$
$$\mathbf{C}_u\in\Re^{6v\times 6v}$$
$$\mathbf{G}_u=\mathbf{G}_s+\mathbf{J}_{sL}{}^T\mathbf{G}_o,\ \mathbf{G}_u\in\Re^{6v}$$

As a result, the Eq. (35) represents the complete dynamic model of the constrained multiple robot systems. All fundamental properties as in (23) are also preserved by this transformation (You, 1994). Note particularly that $(\dot{\mathbf{M}}_u-2\mathbf{C}_u)$ is also a skew-symmetric matrix (see Appendix). However, there exist the coupled relationships between the position variables and the contact force variables. Furthermore, some variables are no longer independent due to the internal and external constraints. From a practical point of view, the dynamic model (35) may not be suitable form for dynamic analysis and hybrid control synthesis.

In what follows, we will derive an appropriate system dynamics which overcomes all the defects mentioned above. Introduce a new generalized coordinate system first such that

$$\mathbf{X}_c=[\mathbf{x}_f{}^T\mathbf{x}_p{}^T]^T\in\Re^{6v} \tag{36}$$

which is completely parameterized by $\mathbf{x}_f=[\Psi_f{}^T\Phi_f{}^T]^T$ and $\mathbf{x}_p=\Phi_p$, where $\mathbf{x}_f\in\Re^{m_t}$ and $\mathbf{x}_p\in\Re^{(6-m)}$. Since the contact surfaces are infinitely rigid, $\mathbf{x}_f=\mathbf{0}_{m_t}$ or equivalently $\Psi_f=\mathbf{0}_{m_c}$ and $\Phi_f=$

$\mathbf{0}_m$. Differentiating (36) with respect to time yields

$$\dot{\mathbf{X}}_c=[\dot{\mathbf{x}}_f{}^T\dot{\mathbf{x}}_p{}^T]^T=[\mathbf{J}_{fl}{}^T\mathbf{J}_{\Phi p}{}^T]^T\dot{\mathbf{q}}_s=\mathbf{J}_c\dot{\mathbf{q}}_s \quad (37)$$

where

$$\dot{\mathbf{x}}_f=\mathbf{J}_{fl}\dot{\mathbf{q}}_s,\ \mathbf{J}_{fl}\in\Re^{m_t\times 6v}$$
$$\dot{\mathbf{x}}_p=\mathbf{J}_p\mathbf{v}_o=\mathbf{J}_{\Phi p}\dot{\mathbf{q}}_s,\ \mathbf{J}_{\Phi p}=\mathbf{J}_p\mathbf{J}_{sL}\in\Re^{(6-m)\times 6v}$$
$$\mathbf{J}_c=[\mathbf{J}_{fl}{}^T\mathbf{J}_{\Phi p}{}^T]^T,\ \mathbf{J}_c\in\Re^{6v\times 6v}$$

In the above formulations, the matrices $\mathbf{J}_{fl}$ and $\mathbf{J}_{\Phi p}$ represent the force-controlled directions and position-manipulated directions, respectively. They have full ranks with $rk(\mathbf{J}_{fl})=m_t$ and $rk(\mathbf{J}_{\Phi p})=6-m$ such that the following relation holds:

$$\mathbf{J}_{\Phi p}\cdot\mathbf{J}_{fl}{}^T=\mathbf{0}\ \ (\text{or}\ \mathbf{J}_{fl}\cdot\mathbf{J}_{\Phi p}{}^T=\mathbf{0}).$$

This implies that $\mathbf{J}_{fl}{}^T$ is a null space of $\mathbf{J}_{\Phi p}$, meaning $rs(\mathbf{J}_{fl}{}^T)\subseteq ns(\mathbf{J}_{\Phi p})$. Using the above relations, we can decompose the space $\Re^{6v}$ into two orthogonal subspaces in the sense that

$$\Re^{6v}=rs(\mathbf{J}_{fl}{}^T)\oplus rs(\mathbf{J}_{\Phi p}{}^T),$$
$$\text{with } rs(\mathbf{J}_{fl}{}^T)\cap rs(\mathbf{J}_{\Phi p}{}^T)=\{\mathbf{0}\}$$

from which a new basis is formed in the $6v$-dimensional spaces.

In case of constrained motions, some variables cannot be specified in arbitrary directions but in the constraint manifold

$$\Delta_{manifold}=\{\mathbf{x}_f\in\Re^{m_t};\ \mathbf{x}_f=\mathbf{0}_{m_t},\ \dot{\mathbf{x}}_f=\mathbf{0}_{m_t}\}.$$

As we shall see, this condition leads to the dimension reduction of the system. Since there exist m_t contact constraints, the overall system has total $(6-m)$ DOF of mobility.

For the further development, we introduce a partitioned identity matrix as

$$\mathbf{E}_{6v}=[\overline{\mathbf{E}}_f : \overline{\mathbf{E}}_p],$$

where $\overline{\mathbf{E}}_f=[\mathbf{E}_{m_t}{}^T : \mathbf{0}^T]^T\in\Re^{6v\times m_t}$ and $\overline{\mathbf{E}}_p=[\mathbf{0}^T : \mathbf{E}_{(6-m)}{}^T]^T\in\Re^{6v\times(6-m)}$. As long as singularities are avoided, some joint-space variables are given below

$$\dot{\mathbf{q}}_s=\mathbf{J}_c^{-1}\dot{\mathbf{X}}_c \quad (38a)$$
$$\ddot{\mathbf{q}}_s=\dot{\mathbf{J}}_c^{-1}\dot{\mathbf{X}}_c+\mathbf{J}_c^{-1}\ddot{\mathbf{X}}_c \quad (38b)$$

Let us introduce the following identities:

$$\mathbf{J}_c^{-T}\mathbf{J}_{fl}{}^T=[\mathbf{J}_{fl}{}^T\mathbf{J}_{\Phi p}{}^T]^{-1}\mathbf{J}_{fl}{}^T=[\mathbf{E}_{m_t}{}^T\ \ \mathbf{0}]^T.$$

By substituting (38a, b) into (35) and multiplying $\mathbf{J}_c{}^{-T}$ on both sides of the resulting equation, the complete system dynamics are given in a set of mixed differential and algebraic equations (DAEs) as follows:

$$\mathbf{M}_c(\mathbf{X}_c;\ \Theta_c)\ddot{\mathbf{X}}_c+\mathbf{C}_c(\mathbf{X}_c,\dot{\mathbf{X}}_c;\ \Theta_c)\dot{\mathbf{X}}_c+\mathbf{G}_c(\mathbf{X}_c;\ \Theta_c)=\mathbf{T}_{cs}-\overline{\mathbf{E}}_f\mathbf{F}_{cl} \quad (39a)$$
$$\mathbf{x}_f=\mathbf{0} \quad (39b)$$

where

$$\mathbf{X}_c=[\mathbf{x}_f{}^T\mathbf{x}_p{}^T]^T,$$
$$\mathbf{M}_c=\mathbf{J}_c{}^{-T}\mathbf{M}_u\mathbf{J}_c{}^{-1}$$
$$\mathbf{C}_c=\mathbf{J}_c{}^{-T}\mathbf{M}_u\dot{\mathbf{J}}_c{}^{-1}+\mathbf{J}_c{}^{-T}\mathbf{C}_u\mathbf{J}_c{}^{-1},$$
$$\mathbf{G}_c=\mathbf{J}_c{}^{-T}\mathbf{G}_u,$$
$$\mathbf{T}_{cs}=\mathbf{J}_c{}^{-T}\mathbf{T}_s.$$

All terms defined above have the corresponding meanings as in (23). Finally, we have derived the unified dynamic model for the entire system. We conclude this section by briefly mentioning the following facts. Since we obtain $\mathbf{x}_f=\mathbf{0}$ in the transformed model, the motion of the entire system is indeed governed by the independent variables $\mathbf{x}_p$. As noted earlier, the position- and force-manipulated subspaces can be easily separated in these formulas. In fact, the dynamic model can be decomposed into two orthogonal subsystems

$$\overline{\mathbf{E}}_p{}^T\mathbf{T}_{cs}=\overline{\mathbf{E}}_p{}^T\mathbf{M}_c\overline{\mathbf{E}}_p\ddot{\mathbf{x}}_p+\overline{\mathbf{E}}_p{}^T\mathbf{C}_c\overline{\mathbf{E}}_p\dot{\mathbf{x}}_p+\overline{\mathbf{E}}_p{}^T\mathbf{G}_c \quad (40a)$$
$$\overline{\mathbf{E}}_f{}^T\mathbf{T}_{cs}=\overline{\mathbf{E}}_f{}^T\mathbf{M}_c\overline{\mathbf{E}}_p\ddot{\mathbf{x}}_p+\overline{\mathbf{E}}_f{}^T\mathbf{C}_c\overline{\mathbf{E}}_p\dot{\mathbf{x}}_p+\overline{\mathbf{E}}_f{}^T\mathbf{G}_c+\mathbf{F}_{cl} \quad (40b)$$

Consequently, the first subsystem represents the reduced-order equations of motion (i. e., purely kinetic differential equations), while the other is concerned with the contact forces.

It is worth realizing that the dynamic model (39a, b) also satisfy the following fundamental properties (You, 1994).

[P6]: $\mathbf{M}_c$ is a positive-definite inertia matrix, and is uniformly bounded by $\underline{\delta}\le\|\mathbf{M}_c\|\le\overline{\delta}$, $\forall\mathbf{X}_c\in\Re^{6v}$, where $\underline{\delta}\,(>0)$ and $\overline{\delta}\,(<\infty)$ are some positive constants.

[P7]: $(\dot{\mathbf{M}}_c-2\mathbf{C}_c)$ is a skew-symmetric matrix, that is, $\mathbf{x}^T(\dot{\mathbf{M}}_c-2\mathbf{C}_c)\mathbf{x}=\mathbf{0}$, $\mathbf{x}\in\Re^{6v}$

Proof: See Appendix for the proof.

[P8]: A part of the system dynamics can be expressed as a linear function in terms of a suitably selected set of system parameters with the vectors $\mathbf{y}$ and $\mathbf{z} \in \Re^{6v}$

$$\mathbf{M}_c(\mathbf{X}_c;\ \Theta_c)\mathbf{z} + \mathbf{C}_c(\mathbf{X}_c, \dot{\mathbf{X}}_c;\ \Theta_c)\mathbf{y} + \mathbf{G}_c(\mathbf{X}_c;\ \Theta_c) = \mathbf{Y}_c(\mathbf{X}_c, \dot{\mathbf{X}}_c, \mathbf{y}, \mathbf{z})\,\Theta_c$$

where $\mathbf{Y}_c \in \Re^{6v \times k_3}$ is regressor matrix, and $\Theta_c \in \Re^{k_3}$ is the system parameter vector.

From a unified point of view, the dynamic model (40a, b) is suitable form for dynamic analysis and hybrid (position and force) control synthesis. The interested reader may find some design examples for hybrid position/force controls of the constrained multiple robots in reference (You, 1994).

6. Conclusions

This paper has provided a unified framework for characterizing the kinematics and dynamical equations of the constrained multiple robot system. Based on an orthogonal complement principle, the combined dynamic model is decomposed into the two orthogonal subsystems: the position and the force subsystems. In fact, a minimal-order governing equation is obtained by utilizing the constraint manifold condition. Furthermore, we can simultaneously and independently manipulate the motion of the object and the contact forces (or the internal grasping forces and the constraint forces). This should be of importance to the dynamics analysis as well as the hybrid (position/force) control synthesis. Due to the closed kinematic chains and the constraint surfaces, it should be noted that the number of degrees of freedom lost in motion is equal to the dimensions of the vector spaces for the contact forces.

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Appendix: Skew-symmetries of the matrices $(\dot{\mathbf{M}}_u - 2\mathbf{C}_u)$ in (35) and $(\dot{\mathbf{M}}_c - 2\mathbf{C}_c)$ in (39).

First, to prove the skew-symmetry of $(\dot{\mathbf{M}}_u - 2\mathbf{C}_u)$, let $\mathbf{N}_u = \dot{\mathbf{M}}_u - 2\mathbf{C}_u$. Then we obtain the following results:

$$\mathbf{x}^T\mathbf{N}_u\mathbf{x}=\mathbf{x}^T\{\dot{\mathbf{M}}_s+2\mathbf{J}_{sL}{}^T\mathbf{M}_o\dot{\mathbf{J}}_{sL}+\mathbf{J}_{sL}{}^T\dot{\mathbf{M}}_o\mathbf{J}_{sL}-2(\mathbf{C}_s+\mathbf{J}_{sL}{}^T\mathbf{M}_o\dot{\mathbf{J}}_{sL}+\mathbf{J}_{sL}{}^T\mathbf{C}_o\mathbf{J}_{sL})\}\mathbf{x}$$
$$=\mathbf{x}^T(\dot{\mathbf{M}}_s-2\mathbf{C}_s)\mathbf{x}+\mathbf{y}^T(\dot{\mathbf{M}}_o-2\mathbf{C}_o)\mathbf{y},\quad \forall\mathbf{x}\in\Re^{6v} \text{ and } \mathbf{y}=\mathbf{J}_{sL}\mathbf{x}$$

where $(\dot{\mathbf{M}}_s-2\mathbf{C}_s)$ and $(\dot{\mathbf{M}}_o-2\mathbf{C}_o)$ are both skew-symmetric matrices. Hence, the matrix $(\dot{\mathbf{M}}_u-2\mathbf{C}_u)$ is also skew-symmetric.

Next, to prove the skew-symmetry of $(\dot{\mathbf{M}}_c-2\mathbf{C}_c)$, let $\mathbf{N}_c=\dot{\mathbf{M}}_c-2\mathbf{C}_c$. Then

$$\mathbf{x}^T\mathbf{N}_c\mathbf{x}=\mathbf{x}^T[2\mathbf{J}_c{}^{-T}\mathbf{M}_u\dot{\mathbf{J}}_c{}^{-1}+\mathbf{J}_c{}^{-T}\dot{\mathbf{M}}_u\mathbf{J}_c{}^{-1}-2(\mathbf{J}_c{}^{-T}\mathbf{M}_u\dot{\mathbf{J}}_c{}^{-1}+\mathbf{J}_c{}^{-T}\mathbf{C}_u\mathbf{J}_c{}^{-1})]\mathbf{x}$$
$$=\mathbf{x}^T\mathbf{J}_c{}^{-T}(\dot{\mathbf{M}}_u-2\mathbf{C}_u)\mathbf{J}_c{}^{-1}\mathbf{x}$$
$$=\mathbf{y}^T(\dot{\mathbf{M}}_u-2\mathbf{C}_u)\mathbf{y},\quad \forall\mathbf{y}=\mathbf{J}_c{}^{-1}\mathbf{x}.$$

Thus the fact that the matrix $(\dot{\mathbf{M}}_u-2\mathbf{C}_u)$ is skew-symmetric confirms that $(\dot{\mathbf{M}}_c-2\mathbf{C}_c)$ is indeed skew-symmetric. The proof is completed.